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Extended Collaboration Pursuing Method for Solving Larger Multidisciplinary Design Optimization Problems

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The collaboration pursuing method is a sampling-based multidisciplinary design optimization method, which does not rely on sensitivity analysis. It was found that the collaboration pursuing method is constrained by the effectiveness of sampling in a design space when solving larger multidisciplinary design optimization problems. Three new modules, that is, discrete sampling, new initialization process, and active design variable control, are developed in this work to extend the collaboration pursuing method's capability in dealing with larger multidisciplinary design optimization problems. Using the collaboration pursuing method with the new modules, called extended collaboration pursuing method, a conceptual aircraft design problem involving structures, aerodynamics, and propulsion is successfully solved. The extended collaboration pursuing method is a promising new multidisciplinary design optimization method to solve larger multidisciplinary design optimization problems with better accuracy and comparable efficiency, when compared with other multidisciplinary design optimization methods.

Nomenclature

f = objective function
$\boldsymbol{G}$ = constraint function associated with $\boldsymbol{g}$
$\boldsymbol{g}$ = vector of inequality constraints
n = number of state parameters
R = range
$\boldsymbol{x}$ = union vector of all design variables, $\{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_i, \ldots, \boldsymbol{x}_n\}$
$\boldsymbol{x}_{cs}$ = vector of design variables shared by the objective and disciplines, $(\{\boldsymbol{x}_{cs1}, \ldots, \boldsymbol{x}_{csi}, \ldots, \boldsymbol{x}_{csn}\})$
$\boldsymbol{x}_i$ = vector of disciplinary design variables of y_i [$\boldsymbol{x}_i \cap \boldsymbol{x}_j$ ($i \neq j$), does not have to be Ø]
Y_i = function associated with y_i
$\boldsymbol{y}$ = vector of state parameters, $\{y_1, \ldots, y_i, \ldots, y_n\}$
$\boldsymbol{y}_{ci}$ = vector of state parameters output from other subsystems to subsystem i, $\{y_j\}$, $j \neq i$
y_i = state parameter/variable i

I. Introduction

MULTIDISCIPLINARY design optimization (MDO) has emerged as a new technology dealing with the design of complex systems involving conflicting design requirements, such as aircraft design. By definition, MDO is "a methodology for the design of complex engineering systems and subsystems that coherently exploits the synergism of mutually interacting phenomena" [1]. MDO aims to optimize a complicated design system involving many coupled subsystems (or disciplines). Finding efficient collaboration among coupled subsystems for achieving the optimum design becomes the most important challenge in MDO.

Research in MDO problem formulation has been active in recent years [2–9]. A broad range of issues and challenges in MDO are reviewed and discussed in [1,10,11]. This work focuses on investigating techniques that facilitate a sampling-based MDO method, called collaboration pursuing method (CPM), in dealing with larger MDO problems with mixed (continuous and discrete) design variables.

The recently developed CPM [12,13] is a sampling-based MDO method, in which effective collaboration among coupled subsystems is achieved by selecting feasible samples with a collaboration model (CM) rather than by using sensitivity analysis. The sampling feature gives the CPM a potential to solve all types of MDO problems, especially the ones with expensive "black-box" functions. Based on sampling, the CPM does not rely on gradient information and it can be readily extended to solve mixed (continuous and discrete) MDO problems. On the other hand, due to the sampling feature, the CPM's efficiency is constrained by the problem dimension when solving larger MDO problems. This paper presents an advancement of the CPM. Three new modules: discrete sampling, new initialization process, and active design variable control are developed to extend CPM's capability for solving larger MDO problems. The CPM with the new modules is referred to as the extended collaboration pursuing method (ECPM) herein. Section II defines a general formulation of MDO problems and reviews the original development of the CPM. Newly developed modules in the ECPM are explained in Sec. III. Results of a conceptual aircraft design problem solved with the ECPM and comparisons between the ECPM and other MDO methods are shown in Sec. IV. Finally, the ECPM is discussed in Sec. V.

II. Optimization Problem and CPM Architecture

According to [11], a general MDO problem can be formulated as

$$\begin{aligned} &\min_x f(\boldsymbol{x}_{cs}, \boldsymbol{y}) \\ &\text{subject to: } y_i = Y_i(\boldsymbol{x}_i, \boldsymbol{x}_{csi}, \boldsymbol{y}_{ci}), \qquad i = 1, \ldots, n \\ &\qquad \boldsymbol{g} = \boldsymbol{G}(\boldsymbol{x}, \boldsymbol{y}) \leq \boldsymbol{0} \end{aligned} \tag{1}$$

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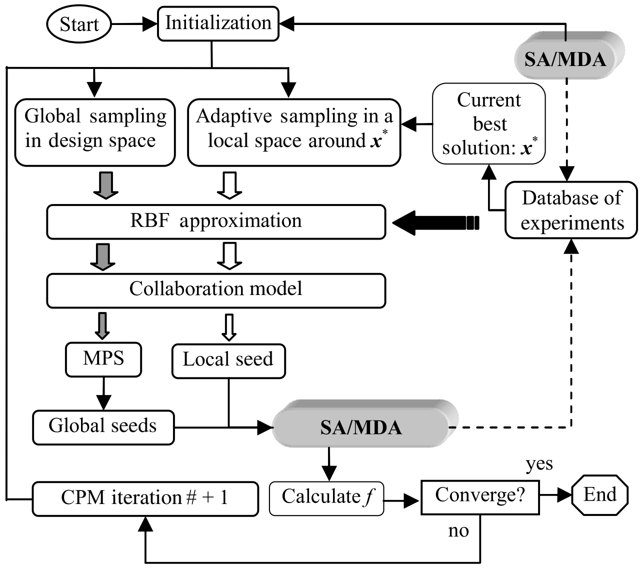

Fig. 1 Architecture of the original collaboration pursuing method [13].

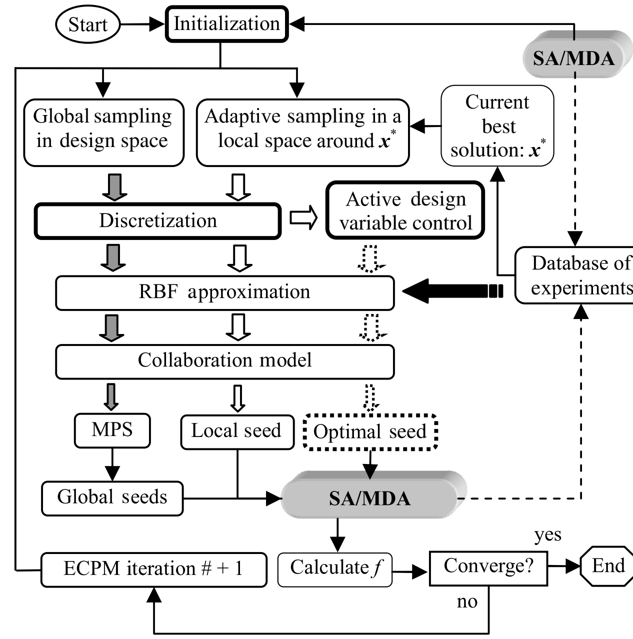

Fig. 2 Architecture of the extended collaboration pursuing method.

In Eq. (1), $\boldsymbol{y}$ is governed by

$$\begin{cases} y_1 = Y_1(\boldsymbol{x}_1, \boldsymbol{y}_{c1}) \\ \cdots \\ y_i = Y_i(\boldsymbol{x}_i, \boldsymbol{y}_{ci}) \\ \cdots \\ y_n = Y_n(\boldsymbol{x}_n, \boldsymbol{y}_{cn}) \end{cases} \tag{2}$$

Equation (2) describes the system analysis (SA) [also called the multidisciplinary analysis (MDA)]. For a fully coupled MDA system (y_i is a function of x_i and the y_j, $i \neq j$, $j = 1, \ldots, n$), $\boldsymbol{y}$ is implicitly dependent of $\boldsymbol{x}$. The solution of Eq. (2) is usually calculated by an iterative procedure. This requires a set of $\boldsymbol{x}$, initial guess of $\boldsymbol{y}$, and convergence criterion determined by a specified accuracy tolerance or a maximum allowed number of iterations. Samples of design variables satisfying SA/MDA in Eq. (2) and constraints $\boldsymbol{g}$ are called feasible samples.

The CPM is a sampling-based method for solving MDO problems [12,13]. The foundations of the CPM were originally developed in [13]. The architecture of the CPM is shown in Fig. 1. The CPM selects from a pool of feasible samples with respect to both SA/MDA and constraints $\boldsymbol{g}$. Then, desirable samples from the selected feasible samples are chosen to be evaluated as new experimental points for optimizing MDO problems. As the number of experimental points increases over CPM iterations, the optimization process moves towards the optimum solution. The CPM's main modules are defined as follows:

As shown in Fig. 1, a collaboration model (CM) was developed to effectively maintain the feasibility of samples with respect to SA/MDA [13], and applied in solving MDO problems [14]. CM reflects both physical and mathematical characteristics of couplings in MDO problems, and models the interdisciplinary discrepancy of coupled state parameters [13]. CM outputs a feasibility distribution of samples with respect to SA/MDA, which can be used to differentiate samples. Effective collaboration among coupled subsystems is realized by selecting samples that are more likely feasible than others, with respect to SA/MDA. Radial basis functions (RBF) defined in Eq. (3) are employed in the CM to approximate the coupled state parameters,

$$\tilde{y}_i(\boldsymbol{x}) = \sum_{e=1}^{E} \alpha_i \|\boldsymbol{x} - \boldsymbol{x}^{(e)}\| \tag{3}$$

where α_i are unknown coefficients calculated by a set of simultaneous linear equations with input data points, $\boldsymbol{x}^{(e)} \in R^m$, $e = 1, 2, \ldots, E$. An adaptive sampling process is applied within the neighborhood of the current best experimental point during the optimization process for achieving local optima. Global optimum solutions are sought when solving MDO problems by applying the mode-pursuing sampling (MPS) method to the CPM framework [15]. The MPS method searches for the global optimum of a black-box function. It is a discriminative sampling method that generates more sample points around the current minimum than other areas, while statistically covering the entire search space [16].

In the original development of the CPM, the initialization process may not be efficient to prepare initial experimental points required by the RBF approximation, because initial experimental points must be feasible, with respect to both SA/MDA [Eq. (2)] and constraints $\boldsymbol{g}$. Also, the CPM is not capable of dealing with larger MDO problems. In this work, new modules (discrete sampling, new initialization process, and active design variable control) are developed in MATLAB® 6.0 [15] and added into the framework of the ECPM. As mentioned earlier, the ECPM is focused on improving the optimization efficiency when solving larger MDO problems. The architecture of the ECPM is shown in Fig. 2, in which highlighted boxes represent the new modules.

III. Extended Collaboration Pursuing Method

The ECPM consists of three new modules: discrete sampling, new initialization, and active design variable control. First, when the number and range of design variables are large, the effectiveness and efficiency of sampling-based methods are degraded, because more samples are needed to effectively cover the entire design variable space. Reducing the dimension of design variables during the optimization process by controlling the number of active design variables could reduce the number of samples to achieve an acceptable level of accuracy and efficiency. The active design variables are defined as the variables whose value will be varied by sampling, rather than being fixed at a certain value. In the ECPM, as shown in Fig. 2, the number of active design variables is controlled by an active design variable control module based on the variation of design variables at the best design solution in two consecutive optimization iterations. Such variation partially reflects the sensitivity information of $\partial f / \partial \boldsymbol{x}$. Like the adaptive sampling process, the active design variable control effectively helps the ECPM pursue local optima and also speed up the optimization process. Second, because only certain accuracy of design variables is meaningful in a real engineering design problem, the sampling

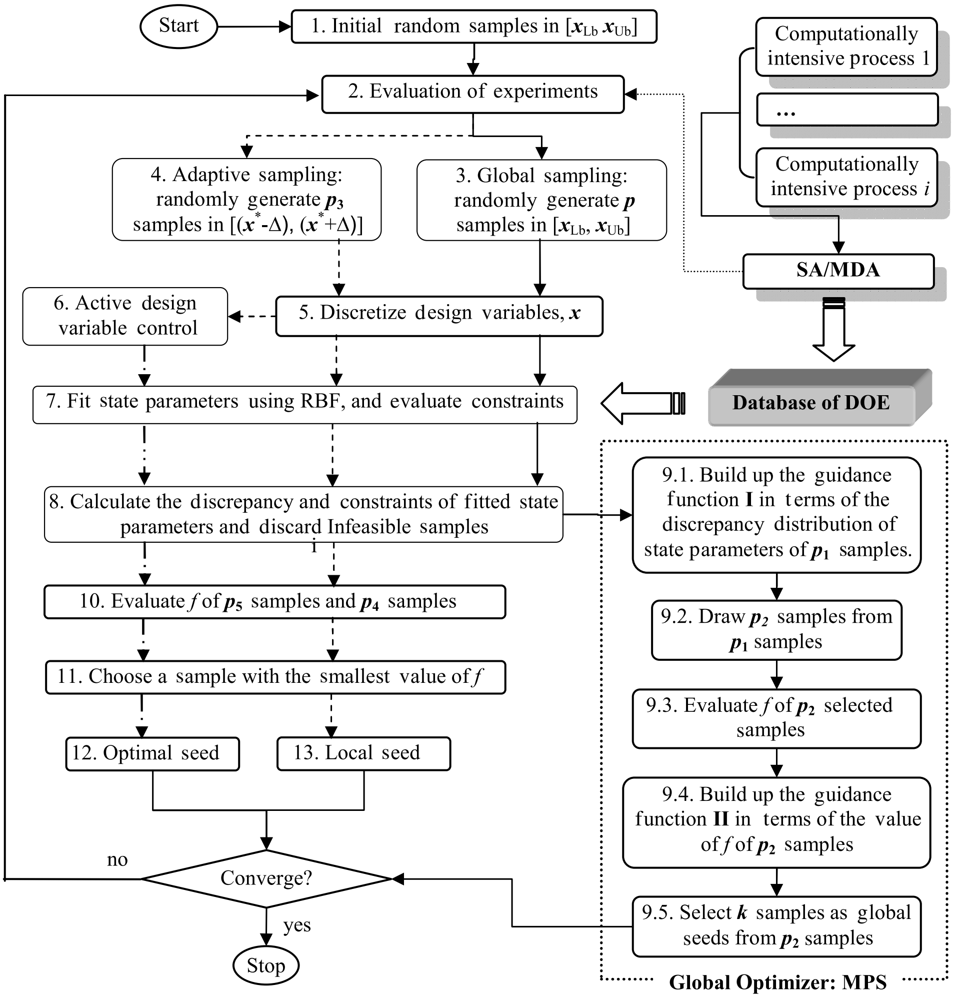

Fig. 3 Flowchart of the extended collaboration pursuing method.

process can be easily modified to discretize continuous design variables based on their accuracy. Discretizing continuous variables can partly alleviate difficulties caused by a large number of design variables, as the number of possible solutions becomes finite. Lastly, a new initialization process is developed to save computational cost for initialization.

In general, all three new modules help the ECPM improve its efficiency. Moreover, both the discrete sampling and active design variable control modules also extend the ECPM's capability for solving larger MDO problems with improved accuracy. The detailed process of the ECPM is elaborated sequentially according to Fig. 3 as in the following subsections (step numbers correspond to box numbers in Fig. 3).

A. New Initialization (Steps 1 and 2)

At the beginning of optimizing MDO problems, the ECPM randomly generates samples in the entire design variable space until several (e.g., four) initial feasible experiments with respect to SA/MDA are obtained. The feasibility of the initial experiments is validated by calling SA/MDA.

In this process, regardless of their feasibility with respect to constraints $\boldsymbol{g}$, initial experiments will be saved in the database of experiments, as long as they satisfy SA/MDA. These samples are used for tuning the RBF approximation, to filter out infeasible samples with respect to SA/MDA. After the initialization process, the initial infeasible experiments with respect to constraints $\boldsymbol{g}$ will be replaced by feasible experiments (selected over ECPM iterations) with respect to both SA/MDA and constraints $\boldsymbol{g}$. Infeasible samples with respect to SA/MDA are discarded and marked to avoid future repeating. In contrast, the original CPM demands initial feasible points with respect to both SA/MDA and constraints $\boldsymbol{g}$ before the approximation is performed. The new initialization module greatly reduces the cost for initialization compared to the initialization process in the original CPM. Detailed analysis of its effectiveness is shown in Sec. IV.D.

B. Sampling (Steps 3 and 4)

Beginning with the first ECPM iteration, two sampling processes take place at the beginning of each ECPM iteration for generating a large number of samples, which will be checked for feasibility by CM and some of which will be selected for optimization.

1) *Global Sampling*—Step 3: A random sampling process employed in the original design space, $[\boldsymbol{x}_{\mathrm{Lb}}, \boldsymbol{y}_{\mathrm{Ub}}]$, where $\boldsymbol{x}_{\mathrm{Lb}}$ and $\boldsymbol{y}_{\mathrm{Ub}}$ are the lower and upper bounds of design variables, respectively, is called global sampling. Global sampling generates p random samples, for example, 10^4, which will be processed by the MPS to search for the global optimum solution at step 9.

2) *Adaptive Sampling*—Step 4: A random sampling process for generating p_3 random samples, for example, 10^4, in the neighborhood of the current best solution $\boldsymbol{x}^*$ is called adaptive sampling. The main idea of adaptive sampling is to have more samples in a small local space around the current best solution. For continuous design variables, the size of a local space for adaptive sampling is determined by

$$\{[\boldsymbol{x}^* - \Delta_1(\boldsymbol{x}_{\mathrm{Ub}} - \boldsymbol{x}_{\mathrm{Lb}})], [\boldsymbol{x}^* + \Delta_1(\boldsymbol{x}_{\mathrm{Ub}} - \boldsymbol{x}_{\mathrm{Lb}})]\} \tag{4}$$

where Δ_1 is a preset ratio between 15 and 30%. For discretized variables, the size of a local space for adaptive sampling can be assigned by

$$[(\boldsymbol{x}^* - Ia_d), (\boldsymbol{x}^* + Ia_d)] \tag{5}$$

where a_d is the accuracy or interval used to discretize continuous variables, and I is an integer, such as 4. The benefit of using adaptive sampling has been demonstrated in [12,13]. The efficiency and accuracy of the CPM were improved significantly as a result.

C. Discrete Sampling (Step 5)

The sample sets given by global sampling and adaptive sampling can be easily discretized so that the ECPM is able to more efficiently solve MDO problems.

Continuous design variable values and the discretized design variable set can be generated by a random sampling procedure in the design variable space, $[\boldsymbol{x}_{\mathrm{Lb}}, \boldsymbol{y}_{\mathrm{Ub}}]$. To make samples meaningful to engineering applications, as well as effective in dealing with larger design problems, discrete sampling is implemented by a one-to-one mapping process between the continuous design variable values and a discretized design variable set. The predetermined accuracy of a design variable discretizes the design variable with finite values within its range. For example, if the range of x_1, for example, $2 \le x_1 \le 3$, and the meaningful accuracy of x_1 with one decimal place, that is, $a_d = 0.1$, are given, x_1 can be discretized to

$$x_{1,d} = \{2.0 \quad 2.1 \quad 2.2 \quad 2.3 \quad 2.4 \quad 2.5 \quad 2.6 \quad 2.7 \quad 2.8 \quad 2.9 \quad 3.0\} \tag{6}$$

In other situations where a design variable is discrete in nature, its discretized variable set consists of all allowable values. For a continuous random sample, its corresponding discrete value is determined by

$$x_1 = x_{1,ds}\{\text{for } \forall\, dz = 1, \ldots, ns, |x_1 - x_{1,ds}| \le |x_1 - x_{1,dz}|\} \tag{7}$$

where ns is the number of discrete values of x_1. For $x_{1,d}$ defined in Eq. (6), if a continuous sample is 2.435677, the discrete value should be 2.4. The meaningful accuracy of design variables is dependent on the physical meaning of design variables and design specifications. For example, in a conceptual aircraft design problem described in Sec. IV, the accuracy of the Mach number is 0.1.

Discrete sampling can help the optimization process reduce the number of possible combinations of design variables. Therefore, the efficiency and capability of the ECPM will be improved with a limited number of random samples. Discrete sampling also extends the applicability of the ECPM to MDO problems with mixed design variables.

D. Active Design Variable Control (Step 6)

The discretized sample set from adaptive sampling is duplicated and passed to the active design variable control[§] module. The design variable value of this duplicated sample set will be adjusted, to reduce the dimension and improve the effectiveness of the samples.

Because the ECPM is fundamentally built on experiments, a part of the sensitivity information could be extracted from two adjacent intermediate best solutions over the past optimization process, in terms of how the objective value varies with design variable values. Using the given sensitivity information, one can estimate the influence of a design variable on the objective in a local space. Intrinsically, the mechanism of active design variable control is based on the sensitivity information $\partial f / \partial \boldsymbol{x}$.

In the active design variable control process, the value of a design variable for all samples (duplicated from adaptive sampling, i.e., p_3 random samples) will be fixed at the current best solution, if this design variable value of the best solution over two consecutive ECPM iterations does not change. Also, each variable has a counter to record the consecutive ECPM iteration number when its value is kept fixed to all samples. Then all design variables can be sorted in an ascending order, in terms of the value of their counter. As all design variables are frozen at the current best solution, a certain number of design variables, which have a larger value of the counter than the remaining design variables, will be reactivated to have random values from sampling. The variable na denotes the number of reactivated design variables in the ECPM. The motivation of reactivating design variables is that the reactivated design variables could be prematurely frozen. As a result, the active design variable control process reduces the dimension of the original MDO problem for sampling.

As shown by a conceptual aircraft design problem solved with the ECPM in Sec. IV, the accuracy and capability of the ECPM in searching for the local optima is significantly enhanced by active design variable control and discrete sampling.

E. RBF Approximation and Optimization (Steps 7–13)

Samples from the global sampling, adaptive sampling, and active design variable control modules are approximated with the RBF based on the database of experimental points. Then these samples are checked by collaboration model with respect to SA/MDA and by constraints $\boldsymbol{g}$, respectively. Infeasible samples are filtered out.

Step 9 employs the MPS to choose feasible points with respect to SA/MDA from the remaining p_1 samples given by the global sampling module. The MPS then searches for the points of the small approximate objective function value from the chosen p_2 feasible points, to pursue the global optimum solution. As a result, several desirable experiments, for example, k experiments, are selected as global seeds at the end of step 9.

Following the dotted lines in Fig. 3, the adaptive sampling at step 4 generates p_3 samples, among which p_4 points satisfy all the constraints and will be evaluated at step 10. Similarly, p_5 samples are left after checking the feasibility of samples generated from the active design variable control at step 6. A sample (having the best approximate objective function value) is chosen from the p_4 samples as a local seed. Another sample (having the best approximate objective function value) is selected as an optimal seed from the p_5 samples.

All selected samples are validated by calling SA/MDA to calculate their values of state parameters $\boldsymbol{y}$ as well as the objective function f. Feasible samples (with respect to SA/MDA and constraints) are saved into the database of experiments to improve the accuracy of the RBF approximation in the next iteration. The ECPM is terminated if there is no further improvement of f after a certain number of consecutive ECPM iterations, such as four iterations. Additional details about the MPS and collaboration model can be found in [12,13].

The collaboration model allows the ECPM to extract useful information in compliance with SA/MDA. Based on the collaboration model, the ECPM selects feasible samples to tune the RBF approximation, and consequently this approximation model expands itself towards the optimum solution of an MDO problem. The discrete sampling, new initialization process, and active design variable control modules make sampling more effective when dealing with larger design problems.

IV. Conceptual Aircraft Design

In this section, a conceptual aircraft design problem applied in [17] is solved with the ECPM. This problem was also solved in [6,7,18]. This design problem has ten design variables, three coupled subsystems/disciplines (structures, aerodynamics, and propulsion), and 12 constraints. The data dependencies of the conceptual aircraft design problem are shown in Fig. 4. The structures subsystem needs the inputs of lift and engine weight from the aerodynamics subsystem and the propulsion subsystem, respectively. In a similar fashion, the aerodynamics subsystem relies on the total weight from the structures subsystem and engine scale factor from the propulsion subsystem, whereas the propulsion subsystem is coupled with the aerodynamics subsystem in terms of drag. As described in [17], some

[§]The active design variable control process can be applied either in the original design variable space, $[\boldsymbol{x}_{\mathrm{Lb}}, \boldsymbol{x}_{\mathrm{Ub}}]$, or in a neighborhood around the current best solution, $\boldsymbol{x}^*$. In this work, it is only applied in the latter situation, as shown in Fig. 3.

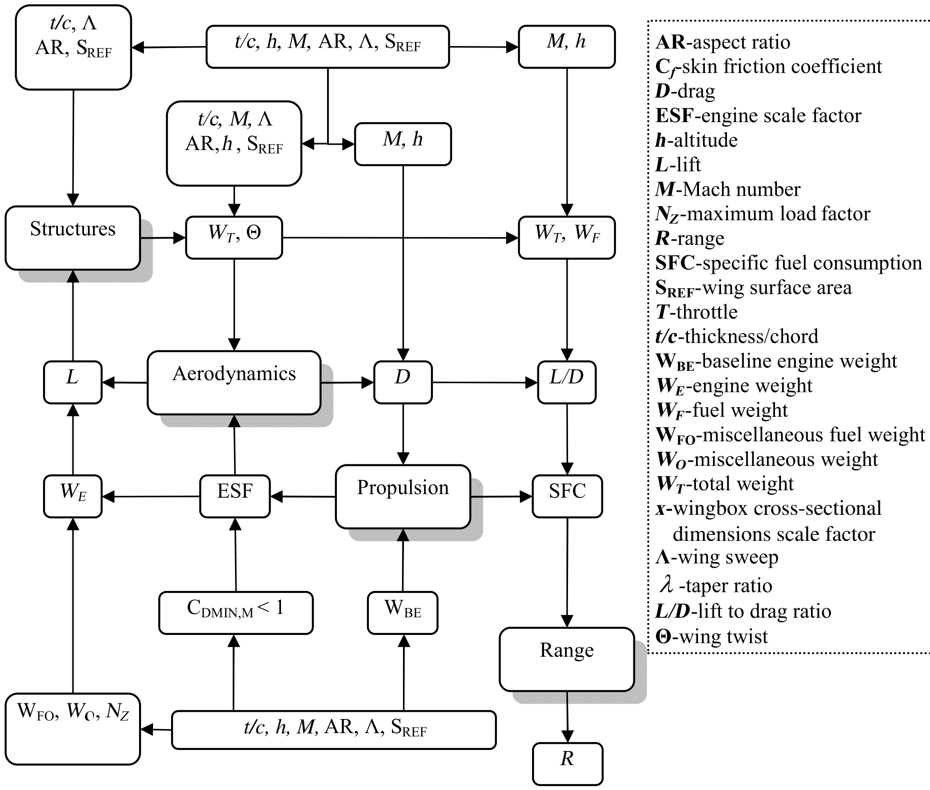

Fig. 4 Data dependencies for aircraft range optimization [17].

typical functions are modeled with polynomial functions to reflect the commonly known relationship between variables, for example, stress falling with the increase of the skin thickness in a wing box. The aircraft design problem aims to maximize the range computed by the Breguet equation. All design variables are listed in Table 1. Additional details about this problem can be found in [12,17].

As shown in Fig. 2, the ECPM applies the new initialization process, both continuous and discrete sampling, and active design variable control modules to maximize the range of the conceptual aircraft. In this work, λ and T are kept as continuous variables, and the remaining variables in Table 1 are discretized based on their assigned accuracy listed in Table 2. The MPS is not used in the ECPM for solving the conceptual aircraft design. Instead, a sample (global seed) with the smallest approximate range value, given by global sampling, is selected as one of the experimental points. For the internal control of the ECPM, the global seed is eliminated if its range value is smaller than that of the optimal seed or local seed. To use the collaboration model in the ECPM, the explicit relations between state parameters and their corresponding variables are shown in Table 3. For example, W_T is an explicit function of W_F, L, W_E, λ, x, t/c, AR, Λ, and S_{REF} in the structures subsystem. Also, it is observed that the union of λ and x, C_f, and T are local design variables in structures, aerodynamics, and propulsion subsystems, respectively, and couplings amongst W_T, D, and ESF dominate the whole system. The implicit relations between the coupled state parameters, W_T, D, and ESF, and their associated variables are listed in Table 4, where "o" signs indicate the dependency based on the implicit relations between state parameters and design variables.

For optimizing the conceptual aircraft design with the ECPM, subroutine codes of SA/MDA, disciplinary analyses (structures, aerodynamics, and propulsion), and range calculations are directly adopted from [17] and embedded in the framework of the ECPM. To compare the results given by the ECPM and the results from [17], some baseline cases are tested based on intermediate and optimum solutions from [17], as shown in Table 5. Results in the ECPM columns in Table 5 are obtained by running the SA/MDA subroutines embedded in the ECPM, rather than the whole ECPM optimization process. Because the values of range are calculated in the ECPM are very close to the values from [17], the ECPM has a basis for comparison with BLISS in [17] for this problem in terms of SA/MDA, disciplinary analyses, and range evaluation. However, one of the constraints, wing twist Θ, of the optimum solution from [17] is violated. Therefore, in the optimization process of the conceptual aircraft design with the ECPM, two types of Θ are applied, that is, $0.96 \le \Theta \le 1.04$ defined in the original formulation in [17], and $0.9049 \le \Theta \le 1.04$ (as shown in Table 5) from the

Table 1 Design variables of the conceptual aircraft design

Variables		Description	Unit	Lower bound	Upper bound
1	λ	Wing taper ratio	N/A	0.1	0.4
2	x	Wingbox cross-sectional dimensions scale factor	N/A	0.75	1.25
3	C_f	Skin friction coefficient	N/A	0.75	1.25
4	T	Throttle setting	N/A	0.1	1
5	t/c	Thickness/chord ratio	N/A	0.01	0.09
6	h	Altitude	ft	30,000	60,000
7	M	Mach number	N/A	1.4	1.8
8	AR	Aspect ratio	N/A	2.5	8.5
9	Λ	Wing sweep	deg	40	70
10	S_{REF}	Wing surface area	ft^2	500	1,500

Table 2 Discretized design variables

Variables	Accuracy
x	0.05
C_f	0.05
t/c	0.01
h, ft	1000
M	0.1
AR	0.1
Λ, deg	1
S_{REF}, ft^2	10

preceding analyses. The complexity of the conceptual aircraft design problem is uncovered by numerical studies in the following subsection.

A. Numerical Studies of the Conceptual Aircraft Design Problem

Numerical studies are conducted based on the extensive enumeration by calling SA/MDA with respect to the original and modified constraints, while the number of random samples is fixed. Ten thousand random samples with respect to the modified constraints ($0.9049 \le \Theta \le 1.04$) are independently generated for

Table 3 Explicit dependency between state parameters and their variables

	State parameters										Local variables				Interdisciplinary variables					
	Structures			Aerodynamics			Propulsion				S		A	P						
	W_F	W_T	Θ	L	D	C_L/C_D	W_E	SFC	ESF	R	λ	x	C_f	T	t/c	h	M	AR	Λ	S_{REF}
Structures (S)																				
W_F	—	—	—	—	—	—	—	—	—	—	—	—	—	—	x	—	—	x	—	x
W_T	x	—	—	x	—	—	x	—	—	—	x	x	—	—	x	—	—	x	x	x
Θ	—	—	—	x	—	—	—	—	—	—	x	x	—	—	—	—	—	x	—	x
σ_1	—	—	—	x	—	—	—	—	—	—	x	x	—	—	x	—	—	x	—	x
σ_2	—	—	—	x	—	—	—	—	—	—	x	x	—	—	x	—	—	x	—	x
σ_3	—	—	—	x	—	—	—	—	—	—	x	x	—	—	x	—	—	x	—	x
σ_4	—	—	—	x	—	—	—	—	—	—	x	x	—	—	x	—	—	x	—	x
σ_5	—	—	—	x	—	—	—	—	—	—	x	x	—	—	x	—	—	x	—	x
Aerodynamics (A)																				
L	—	x	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
D	—	x	x	—	—	—	—	—	x	—	—	—	x	—	x	x	x	x	x	x
C_L/C_D	—	x	x	—	—	—	—	—	x	—	—	—	x	—	x	x	x	x	x	x
$\mathrm{d}p/\mathrm{d}x$	—	—	—	—	—	—	—	—	—	—	—	—	—	—	x	—	—	—	—	—
Propulsion (P)																				
W_E	—	—	—	—	—	—	—	—	x	—	—	—	—	—	—	—	—	—	—	—
SFC	—	—	—	—	—	—	—	—	—	—	—	—	—	x	—	x	x	—	—	—
ESF	—	—	—	—	x	—	—	—	—	—	—	—	—	x	—	—	—	—	—	—
$\bar{T}$	—	—	—	—	—	—	—	—	—	—	—	—	—	x	—	—	—	—	—	—
Temp	—	—	—	—	—	—	—	—	—	—	—	—	—	x	—	x	x	—	—	—
R	x	x	—	—	—	x	—	x	—	—	—	—	—	—	—	x	x	—	—	—

Table 4 Implicit dependency between coupled state parameters and their variables

	State parameters										Local variables				Interdisciplinary variables					
	Structures			Aerodynamics			Propulsion				S		A	P						
	W_F	W_T	Θ	L	D	C_L/C_D	W_E	SFC	ESF	R	λ	x	C_f	T	t/c	h	M	AR	Λ	S_{REF}
Structures (S)																				
W_F	—	—	—	—	—	—	—	—	—	—	—	—	—	—	x	—	—	x	—	x
W_T	—	—	—	—	—	—	—	—	—	—	x	x	o	o	x	o	o	x	x	x
Θ	—	—	—	—	—	—	—	—	—	—	x	x	—	—	—	—	—	x	—	x
σ_1	—	—	—	—	—	—	—	—	—	—	x	x	—	—	x	—	—	x	—	x
σ_2	—	—	—	—	—	—	—	—	—	—	x	x	—	—	x	—	—	x	—	x
σ_3	—	—	—	—	—	—	—	—	—	—	x	x	—	—	x	—	—	x	—	x
σ_4	—	—	—	—	—	—	—	—	—	—	x	x	—	—	x	—	—	x	—	x
σ_5	—	—	—	—	—	—	—	—	—	—	x	x	—	—	x	—	—	x	—	x
Aerodynamics (A)																				
L	—	x	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
D	—	—	—	—	—	—	—	—	—	—	o	o	x	o	x	x	x	x	x	x
C_L/C_D	—	x	x	—	—	—	—	—	x	—	—	—	x	—	x	x	x	x	x	x
$\mathrm{d}p/\mathrm{d}x$	—	—	—	—	—	—	—	—	—	—	—	—	—	—	x	—	—	—	—	—
Propulsion (P)																				
W_E	—	—	—	—	—	—	—	—	x	—	—	—	—	—	—	—	—	—	—	—
SFC	—	—	—	—	—	—	—	—	—	—	—	—	—	x	—	x	x	—	—	—
ESF	—	—	—	—	—	—	—	—	—	—	o	o	o	x	o	o	o	o	o	o
$\bar{T}$	—	—	—	—	—	—	—	—	—	—	—	—	—	x	—	—	—	—	—	—
Temp	—	—	—	—	—	—	—	—	—	—	—	—	—	x	—	x	x	—	—	—
R	x	x	—	—	—	x	—	x	—	—	—	—	—	—	—	x	x	—	—	—

Table 5 Cases from [6,17] and their results given by SA/MDA in the ECPM

Design variables	Case									
	1		2		3		4		5[a]	
λ	0.25		0.14951		0.17476		0.25775		0.38757	
x	1		0.75		0.75		0.75		0.75	
C_f	1		0.75		0.75		0.75		0.75	
T	0.5		0.1676		0.20703		0.15624		0.15624	
t/c	0.05		0.06		0.06		0.06		0.06	
h, ft	45,000		54,000		60,000		60,000		60,000	
M	1.6		1.4		1.4		1.4		1.4	
AR	5.5		4.4		3.3		2.5		2.5	
Λ, deg	55		66		70		70		70	
S_{REF}, ft^2	1,000		1,200		1,400		1,500		1,500	
Constraints	*Method*									
	BLISS	ECPM	BLISS	ECPM	BLISS	ECPM	BLISS	ECPM	BLISS	ECPM
Range	535.79	535.79	1581.67	1581.3	3425.35	3424.7	3961.41	3961.1	3963.98	3963.2
$\sigma_1 \leq 1.09$	——	1.1250	——	1.0553	——	1.0453	——	1.0419	——	1.0696
$\sigma_2 \leq 1.09$	——	1.0833	——	1.0520	——	1.0422	——	1.0358	——	1.0550
$\sigma_3 \leq 1.09$	——	1.0625	——	1.0446	——	1.0362	——	1.0298	——	1.0445
$\sigma_4 \leq 1.09$	——	1.0500	——	1.0384	——	1.0341	——	1.0253	——	1.0371
$\sigma_5 \leq 1.09$	——	1.0417	——	1.0335	——	1.0271	——	1.0219	——	1.0318
$0.96 \leq \Theta \leq 1.04$	——	0.9500	——	0.8961	——	0.9142	——	0.9290	——	0.9049
$0.5 \leq \mathrm{d}p/\mathrm{d}x \leq 1.04$	——	1.000	——	1.0400	——	1.0400	——	1.0400	——	1.0400
ESF ≤ 1.05	——	0.5028	——	0.8023	——	0.5160	——	0.7328	——	0.7328
$\bar{T} - \bar{T}_{\mathrm{UA}} \leq 0$	——	0.1621	——	−0.1905	——	0.3250	——	$-3.04\mathrm{e}^{-5}$	——	$-3.04\mathrm{e}^{-5}$
Temp ≤ 1.02	——	1.000	——	0.8541	——	0.8367	——	0.8367	——	0.8367

[a]Case 5 is the optimum solution given by the BLISS in [6,17].

four times. The average maximum range is 1914.25 nm (nautical miles), which is far from the optimum, 3963.98 nm, given by [17] as shown in the last column of Table 5. Random samples of 10^4, 2×10^4, and 3×10^4 with respect to the original constraints ($0.96 \leq \Theta \leq 1.04$) are also independently generated and each case is executed for four times. It is observed that with more sample points, the average maximum range value improves. However, even with 3×10^4, the average maximum range value is only 1985 nm. It appears that 3×10^4 random samples are not enough to effectively cover the entire design variable space of the conceptual aircraft design problem. This reflects the real challenge of sampling-based optimization methods for solving large-scale MDO problems. The capability of the sampling-based optimization methods is constrained by the sample size, which is related to the number of design variables and their range.

With respect to the original constraints, the effectiveness of applying the adaptive sampling (note: the active design variable control module is not applied in this study) in the ECPM is shown by ten runs with the new initialization process over 35 ECPM iterations, as shown in Table 6. The sample size for global sampling and adaptive sampling is 10^4, Δ_1 equals 0.2, and I is set to 4. At each ECPM iteration, only a local seed and a global seed are selected. As mentioned before, the global seed will be eliminated if its range value is less than that of the local seed over two consecutive ECPM iterations. In comparison to the results with 10^4 random sampling, adaptive sampling effectively lifts up the range value from about 1500 to 3500 nm. It is also observed that the range value obtained by applying the adaptive sampling easily falls in between 2500 and 3500 nm after a number of ECPM iterations. The effectiveness of discrete sampling, the new initialization process, and the active design variable control will be described in Sec. IV.C.

Table 6 Effectiveness of applying the adaptive sampling in the ECPM

Run	Maximum range	Index number of the ECPM iteration[a]
1	3480.071	15
2	3480.071	15
3	3480.071	15
4	3480.071	15
5	3156.942	9
6	3480.071	15
7	3156.942	9
8	3480.071	15
9	3480.071	15
10	3156.942	9
Average	3383.132	13.2

[a]The index number of the CPM iteration indicates when the optimum solution occurred over a total of 35 CPM iterations.

B. Parameter Studies of the ECPM

In this conceptual aircraft design problem, the ECPM involves three parameters, which are Δ_1, I, and na introduced in Secs. III.B and III.D. Δ_1 and I determine the size of a local space around the current best solution, and na is the number of active design variables. The way by which these parameters influence the ECPM's performance is studied with the same initial experiments. The number of random samples (for global sampling, adaptive sampling, and active design variable control, respectively) is 10^4. Four initial experiments listed in Table 7 are infeasible with respect to modified constraints. Also, the maximum number of ECPM iterations is set to 35.

1. *Study on na*

This study is implemented by changing the value of na with respect to two settings of Δ_1 and I. In the first setting, $\Delta_1 = 0.4$ and $I = 5$. In the second setting, $\Delta_1 = 0.3$ and $I = 5$. According to results listed in Table 8, we can roughly conclude that a small na,

Table 7 Initial infeasible experimental points for the parameter studies of the ECPM

Variables	1	2	3	4
λ	0.312689	0.346180	0.192750	0.378284
x	1.15	1.05	1.05	1.10
C_f	1.05	1.2	1.0	0.95
T	0.602455	0.805926	0.611939	0.478591
t/c	0.05	0.07	0.03	0.09
h, ft	52,000	46,000	31,000	57,000
M	1.4	1.6	1.7	1.5
AR	6.1	8.0	6.6	3.4
Λ, deg	45	53	56	61
S_{REF}, ft^2	1,010	660	1,110	1,290
Range, nm	640.061194	337.649591	254.079841	1,634.393249

Table 8 Results of studies on *na*

Case	1 ($na = 1$)		2 ($na = 2$)		3 ($na = 3$)		4 ($na = 4$)	
	Maximum range	Index number of ECPM iteration	Maximum range	Index number of ECPM iteration	Maximum range	Index number of ECPM iteration	Maximum range	Index number of ECPM iteration
$\Delta_1 = 0.4, I = 5$	3960.907	26	3961.077	21	3944.355	26	3827.855	34
$\Delta_1 = 0.3, I = 5$	3958.993	26	3834.720	22	3930.220	16	3849.757	24

Table 9 Results of studies on Δ_1 and *I*

$na = 2$		
Case	Maximum range	Index number of ECPM iteration
1 $\Delta_1 = 0.1, I = 1$	3559.960	19
2 $\Delta_1 = 0.2, I = 2$	3528.280	23
3 $\Delta_1 = 0.3, I = 3$	3835.590	28
4 $\Delta_1 = 0.4, I = 4$	3835.977	21
5 $\Delta_1 = 0.5, I = 5$	3123.022	15

such as $na = 1$, results in a good accuracy of the optimum solution. A relatively large na, for example, $na = 2$ or 3, gives a good efficiency to converge to the optimum solution. As expected, the active design variable control is less effective (the optimum value is low) when na is very large, such as 4. As for this problem, the value of na is suggested to be between 1 and 3 with respect to 10^4 random samples.

2. *Study on Δ_1 and I*

Similarly, the value of na is fixed to be 2 and the study of the influence of Δ_1 and I on the ECPM's performance is conducted, as shown in Table 9. Large values of Δ_1 and I correspond to a big local space around the current best solution, and vice versa. A small local space results in a local optimum solution early, such as in case 1, and a big local space causes adaptive sampling to be less effective, such as in case 5. In this problem, the value of Δ_1 is suggested between 0.2 and 0.4, and the value of I is recommended between 3 and 4 with respect to 10^4 random samples.

C. Optimization Results with Respect to the Modified Constraints

This conceptual aircraft design problem was solved previously with the all-in-one formulation and the all-in-one with the response surface method in [18], BLISS method in [6,17,18], and BLISS with response surfaces in [7]. The results reported in [6,7,17,18] are listed in Table 10. Based on the number of subsystem analyses with the all-in-one method, it seems that each SA takes four iterations to converge, and one iteration costs three subsystem analyses, that is, structural, aerodynamic, and propulsion analyses. Because the subroutine codes of the SA and subsystem analyses applied in the ECPM are identical to codes used in [6,7,17,18], the method of calculating the number of subsystem analyses in [6,7,17,18] is applied to the ECPM for the conceptual aircraft design problem.

According to Table 5 with the optimum solution from [17], the constraint of Θ is modified to be $0.9049 \leq \Theta \leq 1.04$. The remaining constraints are still the same as the original. Based on the parametric studies of Δ_1, I, and na, the values of these parameters are specified as follows:

$$\Delta_1 = 0.3 \qquad I = 4 \qquad na = 2 \tag{8}$$

The optimization process of the ECPM is run based on five cases with different initial infeasible experiments with respect to constraints, as shown in Table 11. The number of random samples (for global sampling, adaptive sampling, and active design variable control, respectively) is 10^4. The number of initial experiments is four and these initial points are listed in Table 11 above the optimal solution in each row, which corresponds to each case. The optimum solutions of all five cases satisfy the constraints (results omitted). Each case is executed six times, and the results are shown in Table 12. Because of the statistical nature of random sampling, the ECPM could have different optimum solutions with the same initial experiments. Based on the average computational cost of each case in Table 12 and the costs listed in Table 10, the ECPM is more efficient than the all-in-one and all-in-one/RS when solving this problem. The ECPM is also competitively efficient against the BLISS, BLISS/RS1, and BLISS/RS2 when solving the conceptual aircraft design problem.

The distribution of experimental points of case 4 in Table 11 is plotted in Fig. 5. We can see that the ECPM started with four initial infeasible experiments, as shown by the "∇" sign, with a very poor range value. At the very beginning of the optimization process, the global seeds marked with the "*" sign lead the optimization process. Then the local seed marked by the "□" sign takes over the leading role after several ECPM iterations, while it is limited under about 3500 nm. Finally, the optimal seeds marked with the "◇" sign make the optimization process converge towards the optimum solution. The convergence of range over iterations is shown in Fig. 6 for case 4.

D. Effectiveness Analysis of New Modules

Based on the comparison between the results in Table 12, when the active design variable control module is applied, and the results listed in Table 6, in which the active design variable control module is not applied, the active design variable control process effectively reduces the dimension of the optimization problem in a local space and improves the optimum solution from about 3383 to about 3926 nm (average value in Table 12).

The original CPM has a high cost in preparing initial feasible experiments with respect to SA/MDA and constraints based on a random sampling process. In the conceptual aircraft design problem, the computational costs for generating four feasible experiments are

Table 10 Results of the conceptual aircraft design given from [6,7,17,18]

Case	Initial objective	Initial max. constraint value	Final objective	Final max. constraint value	Computational effort	
					Number of SA	Number of subsystem analyses[a]
All-in-one	535.79	−0.162	3964.19	$1.0e^{-8}$	119	$119 \times 4 \times 3 = 1428$
All-in-one/RS[b]	535.79	−0.162	3974.84	0.0013	72	864
BLISS	535.79	−0.162	3964.07	$192e^{-5}$	7	491
BLISS/RS1[c]	535.79	−0.162	3961.50	0.0	17	354
BLISS/RS2[d]	535.79	−0.162	3964.12	0.0	12	1097

[a]Subsystem means subsystem.
[b]The All-in-one/RS is a sequential approximation-based all-in-one optimization strategy that involves the use of response surface model for approximation evaluations of the design objective and constraint functions.
[c]The BLISS/RS1 builds up a response surface to approximate the objective function and constraints in **Z** space based on the data from SA/MDA.
[d]The BLISS/RS2 builds up a response surface to approximate the objective function and constraints in **Z** space based on the data from the BB optimization.

Table 11 Results given by the ECPM with respect to the modified constraints[a]

Case	Design variables												No. of SA when range* occurs	Index no.of the ECPM iteration when range* occurs	Number of subproblem analyses
		λ	x	C_f	T	t/c	h	M	AR	Λ	S_{REF}	Range			
1	Initial design variables	0.385039	0.85	1.05	0.537384	0.08	53,000	1.6	2.6	65	940	1384	45	21	540 ($45 \times 4 \times 3$)
		0.284630	1.15	1.20	0.764387	0.02	42,000	1.8	8.0	52	1390	244			
		0.117367	0.95	1.15	0.108875	0.02	36,000	1.5	6.1	48	700	196			
		0.104582	1.10	0.95	0.938633	0.05	43,000	1.7	5.7	46	1170	286			
	Optimum	0.370513	0.75	0.75	0.156244	0.06	60,000	1.4	2.5	70	1490	3946.9			
2	Initial design variables	0.241243	1.20	1.10	0.117881	0.08	44,000	1.5	7.3	63	1060	419	40	18	480 ($40 \times 4 \times 3$)
		0.143925	1.20	1.00	0.106188	0.04	57,000	1.4	7.1	52	1290	911			
		0.389819	0.80	0.85	0.453128	0.02	47,000	1.7	5.9	50	1460	433			
		0.196969	0.80	1.00	0.719632	0.03	53,000	1.7	7.8	60	1070	523			
	Optimum	0.128208	0.75	0.75	0.156210	0.06	60,000	1.4	2.5	70	1500	3958.6			
3	Initial design variables	0.106628	0.90	1.00	0.962897	0.04	44,000	1.5	4.1	67	530	437	36	17	432 ($36 \times 4 \times 3$)
		0.142364	0.85	1.15	0.982815	0.05	35,000	1.5	4.7	58	700	270			
		0.300568	1.15	1.00	0.932244	0.06	43,000	1.7	3.8	67	1480	529			
		0.259309	1.15	0.80	0.687146	0.01	41,000	1.5	3.2	55	880	337			
	Optimum	0.270599	0.75	0.75	0.156240	0.06	60,000	1.4	2.5	70	1500	3961.3			
4	Initial design variables	0.199809	0.80	1.10	0.790933	0.05	47,000	1.8	2.8	54	1100	602	26	13	312 ($26 \times 4 \times 3$)
		0.364464	1.00	0.80	0.113399	0.02	48,000	1.5	4.0	58	1280	451			
		0.174475	1.20	1.20	0.194790	0.01	39,000	1.8	4.5	52	940	155			
		0.353402	1.10	0.95	0.705562	0.03	53,000	1.5	7.4	48	700	554			
	Optimum	0.325759	0.75	0.75	0.156231	0.06	60,000	1.4	2.5	70	1500	3962.2			
5	Initial design variables	0.241510	1.00	1.20	0.746822	0.06	33,000	1.7	6.8	48	590	245	42	21	504 ($42 \times 4 \times 3$)
		0.276891	1.20	1.25	0.959758	0.04	50,000	1.4	7.3	55	1160	347			
		0.171966	0.95	1.15	0.995564	0.03	35,000	1.7	3.9	65	720	269			
		0.233039	0.80	0.80	0.814310	0.09	44,000	1.4	4.0	64	1000	677			
	Optimum	0.351410	0.75	0.75	0.156213	0.06	60,000	1.4	2.5	70	1500	3962.6			

[a]All initial experimental points are infeasible with respect to the constraints, and "*" indicates the optimum solution.

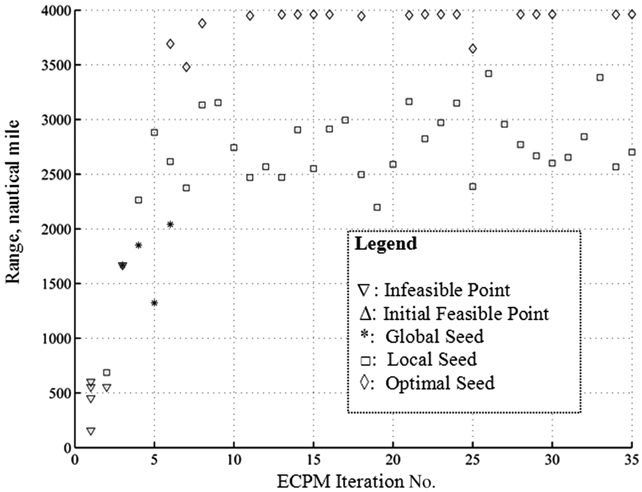

Fig. 5 Distribution of experimental points over 35 ECPM iterations of case 4 in Table 11.

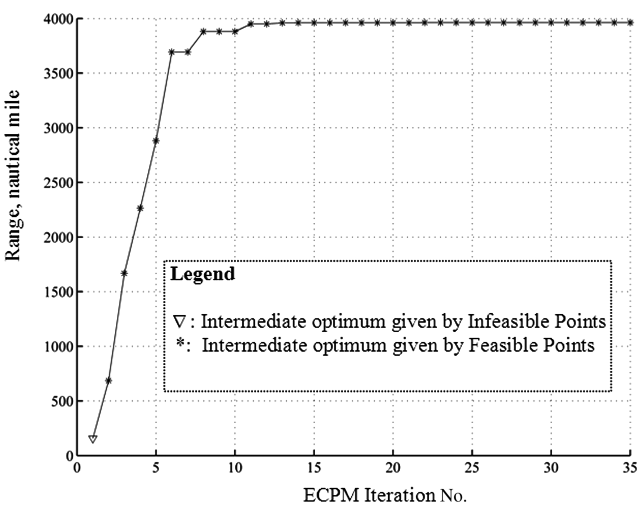

Fig. 6 Convergence of range over 35 ECPM iterations of case 4 in Table 11.

Table 13 Results of Case 4 in Table 11 based on continuous sampling[a]

	Run				Average
	1	2	3	4	
Range*	3497	2966	3497	2966	3231.5
No. of SA when range* occurs	54	38	54	38	46
Index no. of the ECPM iteration when range* occurs	25	17	25	17	21

[a]"*" indicates the optimum solution.

recorded from ten independent runs with respect to the modified constraints, using the original initialization method. It is found that it takes an average of 120 SAs for initialization with the original initialization method, whereas the average total number of SAs for optimization in Table 12 is about 40 with the new initialization process. Apparently, *the new initialization strategy greatly improves the efficiency of ECPM*. The motivation for initializing ECPM with experiments that are infeasible with respect to constraints is that these infeasible experiments still reflect the mathematical relation of coupled state parameters. Thus, the infeasible experiments can be used for initialization to improve the RBF approximation model.

To show the effectiveness of discrete sampling, case 4 in Table 11 is optimized with the ECPM based on continuous sampling as shown in Table 13 (other conditions applied in the ECPM for this study are the same as specified for solving case 4 in Table 11). The average optimal range value is about 3231 nm, which is much less than the optimal range of case 4 in Tables 11 and 12. *Thus, given a fixed total number of samples, the discrete sampling significantly improves the solution quality, as compared with continuous sampling*.

E. Optimization Results with Respect to the Original Constraints

To optimize the conceptual aircraft design problem with respect to the original constraints, $0.96 \le \Theta \le 1.04$, the parameters in the ECPM are set as follows:

$$\Delta_1 = 0.2 \qquad I = 4 \qquad na = 4 \tag{9}$$

The value of na is increased because a greater number of design variables can increase the diversity of samples, so that more feasible samples could survive with respect to the tightened constraint of Θ. As a result, the chance to reach the real optimum is expected to be higher. The number of random samples (for global sampling, adaptive sampling, and active design variable control, respectively)

Table 12 Results of multiple runs of cases in Table 11[a]

Run		Case 1	Case 2	Case 3	Case 4	Case 5
1	Range*	3946.9	3958.6	3961.3	3962.2	3962.6
	No. of SA	45	40	36	26	42
	Number of subsystem analyses	540(45 × 4 × 3)	480(40 × 4 × 3)	432(36 × 4 × 3)	312(26 × 4 × 3)	504(42 × 4 × 3)
2	Range*	3946.9	3958.6	3914.4	3962.2	3958.1
	No. of SA	45	40	27	26	62
	Number of subsystem analyses	540(45 × 4 × 3)	480(40 × 4 × 3)	324(27 × 4 × 3)	312(26 × 4 × 3)	744(62 × 4 × 3)
3	Range*	3946.9	3960.7	3863.5	3962.2	3836.1
	No. of SA	45	58	46	26	46
	Number of subsystem analyses	540(45 × 4 × 3)	696(8 × 4 × 3)	552(46 × 4 × 3)	312(26 × 4 × 3)	552(46 × 4 × 3)
4	Range*	3946.9	3945.9	3898.8	3944.7	3835.0
	No. of SA	45	34	60	43	36
	Number of subsystem analyses	540(45 × 4 × 3)	408(34 × 4 × 3)	720(60 × 4 × 3)	516(43 × 4 × 3)	432(36 × 4 × 3)
5	Range*	3946.9	3963.1	3930.5	3960.1	3959.4
	No. of SA	45	33	28	55	39
	Number of subsystem analyses	540(45 × 4 × 3)	396(33 × 4 × 3)	336(28 × 4 × 3)	660(55 × 4 × 3)	468(39 × 4 × 3)
6	Range*	3959.9	3883.3	3639.8	3960.3	3913.3
	No. of SA	49	39	29	29	33
	Number of subsystem analyses	588(49 × 4 × 3)	468(39 × 4 × 3)	348(29 × 4 × 3)	348(29 × 4 × 3)	396(33 × 4 × 3)
Average	Range*	3949.1	3945.0	3868.05	3958.6	3910.8
	No. of SA	45.7	40.7	37.7	34.2	43
	Number of subsystem analyses	548.4(45.7 × 4 × 3)	488(40.7 × 4 × 3)	452.4(37.7 × 4 × 3)	410.4(34.2 × 4 × 3)	516(43 × 4 × 3)

[a]"*" indicates the optimum solution.

Table 14 Results given by the ECPM with respect to original constraints[a]

Case	Ranges of initial experimental points, nm	Range*, nm	No. of SA when range* occurs	Index no. of the ECPM iteration when range* occurs	Number of subsystem analyses
1	373.8, 800.2, 380.9, 452.2	3910.6	66	33	$792(66 \times 4 \times 3)$
2	1005.6, 438.7, 186.3, 155.5	3830.3	33	16	$396(33 \times 4 \times 3)$
3	766.1, 223.5, 682.8, 262.7	3806.2	66	34	$792(66 \times 4 \times 3)$
4	530.6, 1015.9, 348.6, 445.1	3834.5	61	29	$732(61 \times 4 \times 3)$
5	196.5, 740.8, 383.6, 1060.5	3942.5	62	31	$744(62 \times 4 \times 3)$
6	397.9, 800.6, 374.7, 300.1	3924.4	39	18	$468(39 \times 4 \times 3)$
7	873.7, 874.0, 472.3, 571.9	3958.4	48	24	$576(48 \times 4 \times 3)$
8	290.6, 508.1, 405.7, 250.7	3792.2	65	31	$780(65 \times 4 \times 3)$
Average	——	3893.767	55	27	$660(55 \times 4 \times 3)$

[a]"*" indicates the optimum solution.

is still 10^4. The optimization process is independently executed eight times with different initial infeasible experiments with respect to the constraints $\boldsymbol{g}$. The optimum solution and the computational cost of each case are reported in Table 14. All original data are available in the Appendix. For case 6, the distribution of experimental points is plotted in Fig. 7, and the convergence of the range is shown in Fig. 8. By referring to the constraint value of the optimum solution of each case, as shown in the Appendix, the conceptual aircraft design problem is a constrained optimization problem with respect to Θ. Clearly, the computational cost is increased in Table 14 compared to the cost required by the ECPM with respect to the modified constraints in Table 12. This occurs because the constraint of Θ is tightened and active at the optimum solution. Thus, the feasible region in the design space is narrowed down. Consequently, the number of effective samples is smaller than that with respect to the modified constraints.

V. Discussion

Given an allowed number of random samples, the effectiveness of adaptive sampling is dependent on the size of the local region around the current best solution, that is, Δ_1 or I. The values of Δ_1 or I also depend on the number of design variables. For a small number of design variables, more samples will cover a wider range of design variables effectively. In practice, the values of Δ_1 or I can be dynamically adjusted based on feedback from the past ECPM iterations. For example, if the objective function value cannot be improved by the adaptive sampling, the value of Δ_1 or I could be reduced.

In the active design variable control process, the number of reactivated design variables depends on the random sample size, and the sampling region for active design variable control defined in Eqs. (4) and (5). In general, a large number of random samples and a small region defined by Δ_1 or I allow more active design variables. In practice, the value of na should be specified by users. Feedback from previous ECPM iterations in the optimization process can also be used for adjusting the value of na. For example, if the objective function value does not improve over a certain number of consecutive ECPM iterations (note: other parameters kept fixed, e.g., Δ_1 or I), the value of na could be reduced to make sampling more effective.

In the application of the conceptual aircraft design problem, it has been shown that 3×10^4 random samples are not sufficient to reach even close to the optimum. Consequently, global seeds selected by the MPS from samples given by the global sampling will not be effective in searching for the optimum. Therefore, the MPS is not used for solving the conceptual aircraft design problem, but it could be used in a subregion of the design variable space. This shows again that large-scale MDO problems introduce new challenges in searching for the global optimum solution. Also, the ECPM is based on random sampling, which brings randomness to the optimum solution and efficiency, as shown in Tables 11, 12, and 14. Applying more efficient sampling strategies to the ECPM, such as Latin hypercube sampling, might further improve the efficiency and effectiveness of the ECPM.

VI. Conclusions

The extended collaboration pursuing method (ECPM), developed in this work, has been successfully applied to solve a conceptual aircraft design problem. The new initialization strategy, discrete sampling, and active design variable control modules are demonstrated to function effectively in the ECPM, when coping with difficulties arising from larger design problems. The new initialization strategy allows infeasible points to be used for RBF approximations and thus increase the efficiency dramatically. Discrete sampling helps to improve the effectiveness of sampling and also can be extended for solving mixed variable problems. The

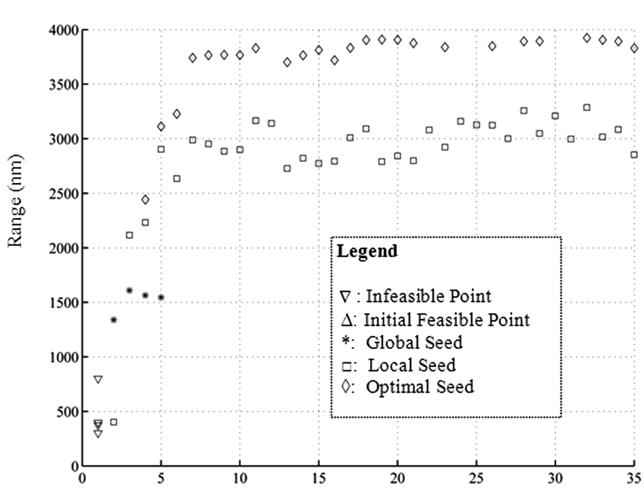

Fig. 7 Distribution of experimental points over 35 ECPM iterations of case 6 in Table 14.

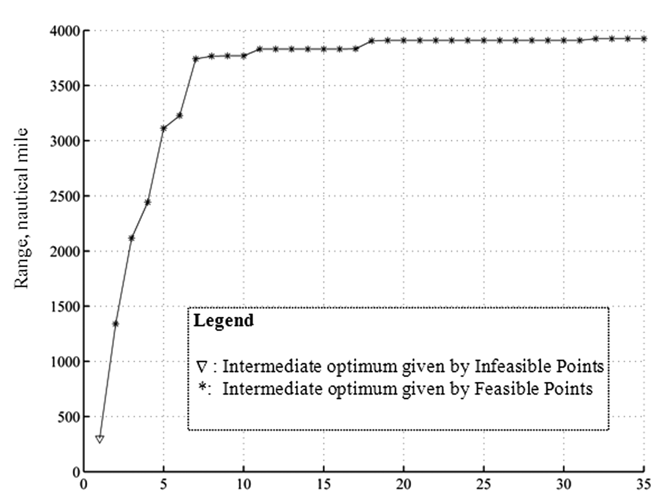

Fig. 8 Convergence of range over 35 ECPM iterations of case 6 in Table 14.

Table A1 Results of the conceptual aircraft optimization in Table 14[a]

Case	Initial infeasible experimental points[b]				Optimal design variables	Optimal state parameters	Constraints at the optimum solution
1	$x1 = 0.234500$	$x1 = 0.355195$	$x1 = 0.254663$	$x1 = 0.155419$			
	$x2 = 1.000000$	$x2 = 1.000000$	$x2 = 1.000000$	$x2 = 0.950000$	$x1 = 0.120449$		$c1 = 1.003528$
	$x3 = 0.800000$	$x3 = 0.900000$	$x3 = 0.900000$	$x3 = 1.000000$	$x2 = 0.750000$	$y1 = 44,403.824060$	$c2 = 1.008828$
	$x4 = 0.763701$	$x4 = 0.318047$	$x4 = 0.446012$	$x4 = 0.900598$	$x3 = 0.750000$	$y2 = 5,442.048306$	$c3 = 1.009050$
	$x5 = 0.040000$	$x5 = 0.070000$	$x5 = 0.010000$	$x5 = 0.070000$	$x4 = 0.156201$	$y3 = 0.718264$	$c4 = 1.008405$
	$x6 = 46,000$	$x6 = 56,000$	$x6 = 46,000$	$x6 = 42,000$	$x5 = 0.060000$	$y5 = 18,832.647098$	$c5 = 1.007652$
	$x7 = 1.600$	$x7 = 1.800$	$x7 = 1.500$	$x7 = 1.500$	$x6 = 60,000.00$	$y6 = 0.960814$	$c6 = 0.960814$
	$x8 = 7.300$	$x8 = 6.400$	$x8 = 4.10$	$x8 = 2.500$	$x7 = 1.400000$	$y7 = 44,403.824060$	$c7 = 1.040000$
	$x9 = 42.00$	$x9 = 57.00$	$x9 = 49.00$	$x9 = 46.000$	$x8 = 2.500000$	$y8 = 8.159395$	$c8 = 0.718264$
	$x10 = 1,500$	$x10 = 1,240$	$x10 = 1,330$	$x10 = 1,140.00$	$x9 = 70.00000$	$y9 = 0.923953$	$c9 = 0.836745$
	$f1 = 373.756197$		$f2 = 800.160567$		$x10 = 1,470.00$	$y10 = 9,240.721209$	$c10 = -0.278e^{-3}$
	$f3 = 380.861400$		$f4 = 452.226646$				
2	$x1 = 0.287451$	$x1 = 0.124758$	$x1 = 0.282065$	$x1 = 0.361815$			
	$x2 = 1.050$	$x2 = 1.000000$	$x2 = 1.150000$	$x2 = 0.850000$	$x1 = 0.170562$		$c1 = 0.998161$
	$x3 = 1.050000$	$x3 = 1.000000$	$x3 = 0.850000$	$x3 = 0.800000$	$x2 = 0.800000$	$y1 = 49,693.232404$	$c2 = 1.002569$
	$x4 = 0.342438$	$x4 = 0.119520$	$x4 = 0.362931$	$x4 = 0.147002$	$x3 = 0.750000$	$y2 = 5,554.706172$	$c3 = 1.003350$
	$x5 = 0.090000$	$x5 = 0.040000$	$x5 = 0.020000$	$x5 = 0.080000$	$x4 = 0.154891$	$y3 = 0.739335$	$c4 = 1.003363$
	$x6 = 58,000.00$	$x6 = 48,000.00$	$x6 = 30,000.00$	$x6 = 32,000.00$	$x5 = 0.060000$	$y5 = 19,350.552345$	$c5 = 1.003182$
	$x7 = 1.500000$	$x7 = 1.700000$	$x7 = 1.600000$	$x7 = 1.700000$	$x6 = 60,000.00$	$y6 = 0.960005$	$c6 = 0.960005$
	$x8 = 5.300000$	$x8 = 4.000000$	$x8 = 8.200000$	$x8 = 7.300000$	$x7 = 1.400000$	$y7 = 49,693.232404$	$c7 = 1.040000$
	$x9 = 50.00000$	$x9 = 50.00000$	$x9 = 54.00000$	$x9 = 55.00000$	$x8 = 2.500000$	$y8 = 8.946150$	$c8 = 0.739335$
	$x10 = 910.000$	$x10 = 560.000$	$x10 = 880.000$	$x10 = 1,200.00$	$x9 = 70.00000$	$y9 = 0.924571$	$c9 = 0.836745$
	$f1 = 1005.622323$		$f2 = 438.708252$		$x10 = 1,500.00$	$y10 = 9,525.570215$	$c10 = -0.8664e^{-2}$
	$f3 = 186.349524$		$f4 = 155.519235$				
3	$x1 = 0.280412$	$x1 = 0.345357$	$x1 = 0.152164$	$x1 = 0.286179$			
	$x2 = 1.100000$	$x2 = 0.800000$	$x2 = 0.850000$	$x2 = 0.950000$	$x1 = 0.155668$		$c1 = 0.992851$
	$x3 = 1.000000$	$x3 = 1.100000$	$x3 = 0.850000$	$x3 = 0.900000$	$x2 = 0.800000$	$y1 = 49,202.759320$	$c2 = 0.998576$
	$x4 = 0.550635$	$x4 = 0.910018$	$x4 = 0.321407$	$x4 = 0.840457$	$x3 = 0.750000$	$y2 = 5,480.668670$	$c3 = 1.000186$
	$x5 = 0.020000$	$x5 = 0.060000$	$x5 = 0.020000$	$x5 = 0.080000$	$x4 = 0.156197$	$y3 = 0.723381$	$c4 = 1.000750$
	$x6 = 53,000.00$	$x6 = 31,000.00$	$x6 = 53,000.00$	$x6 = 34,000.000$	$x5 = 0.060000$	$y5 = 19,004.700585$	$c5 = 1.000959$
	$x7 = 1.600000$	$x7 = 1.700000$	$x7 = 1.600000$	$x7 = 1.400000$	$x6 = 60,000.00$	$y6 = 0.964877$	$c6 = 0.964877$
	$x8 = 3.500000$	$x8 = 6.800000$	$x8 = 5.200000$	$x8 = 7.100000$	$x7 = 1.400000$	$y7 = 49,202.759320$	$c7 = 1.040000$
	$x9 = 42.00000$	$x9 = 50.00000$	$x9 = 43.00000$	$x9 = 50.000000$	$x8 = 2.500000$	$y8 = 8.977510$	$c8 = 0.723381$
	$x10 = 530.000$	$x10 = 1,140.00$	$x10 = 770.000$	$x10 = 1,220.000$	$x9 = 70.00000$	$y9 = 0.923955$	$c9 = 0.836745$
	$f1 = 766.077219$		$f2 = 223.514664$		$x10 = 1,480.00$	$y10 = 9,309.854407$	$c10 = -0.305e^{-3}$
	$f3 = 682.839907$		$f4 = 262.717500$				
4	$x1 = 0.126724$	$x1 = 0.204810$	$x1 = 0.250837$	$x1 = 0.260782$			
	$x2 = 0.950000$	$x2 = 1.250000$	$x2 = 1.150000$	$x2 = 0.900000$	$x1 = 0.133390$		$c1 = 0.987758$
	$x3 = 1.000000$	$x3 = 0.850000$	$x3 = 0.850000$	$x3 = 0.750000$	$x2 = 0.800000$	$y1 = 49,625.267200$	$c2 = 0.995352$
	$x4 = 0.234876$	$x4 = 0.174619$	$x4 = 0.175360$	$x4 = 0.289290$	$x3 = 0.750000$	$y2 = 5,554.617762$	$c3 = 0.997831$
	$x5 = 0.020000$	$x5 = 0.060000$	$x5 = 0.040000$	$x5 = 0.050000$	$x4 = 0.156218$	$y3 = 0.733044$	$c4 = 0.998897$
	$x6 = 48,000.00$	$x6 = 54,000.00$	$x6 = 41,000.00$	$x6 = 45,000.00$	$x5 = 0.060000$	$y5 = 19,350.552345$	$c5 = 0.999432$
	$x7 = 1.500000$	$x7 = 1.500000$	$x7 = 1.500000$	$x7 = 1.600000$	$x6 = 60,000.00$	$y6 = 0.967819$	$c6 = 0.967819$
	$x8 = 4.100000$	$x8 = 4.700000$	$x8 = 6.000000$	$x8 = 6.900000$	$x7 = 1.400000$	$y7 = 49,625.267200$	$c7 = 1.040000$
	$x9 = 51.00000$	$x9 = 60.00000$	$x9 = 43.00000$	$x9 = 52.000000$	$x8 = 2.500000$	$y8 = 8.934056$	$c8 = 0.733044$
	$x10 = 1,160.00$	$x10 = 1,230.00$	$x10 = 520.000$	$x10 = 890.0000$	$x9 = 70.00000$	$y9 = 0.923946$	$c9 = 0.836745$
	$f1 = 530.557297$		$f2 = 1,015.889269$		$x10 = 1,500.00$	$y10 = 9,440.483839$	$c10 = -0.17e^{-3}$
	$f3 = 348.634034$		$f4 = 445.106975$				

(continued)

Table A1 Results of the conceptual aircraft optimization in Table 14[a] (Continued)

Case	Initial infeasible experimental points[b]				Optimal design variables	Optimal state parameters	Constraints at the optimum solution
5	$x1 = 0.326506$	$x1 = 0.370768$	$x1 = 0.108218$	$x1 = 0.213221$			
	$x2 = 1.000000$	$x2 = 0.900000$	$x2 = 1.200000$	$x2 = 0.750000$	$x1 = 0.114856$		$c1 = 1.003345$
	$x3 = 0.900000$	$x3 = 0.800000$	$x3 = 0.750000$	$x3 = 0.750000$	$x2 = 0.750000$	$y1 = 44,756.107036$	$c2 = 1.008948$
	$x4 = 0.573440$	$x4 = 0.480827$	$x4 = 0.764863$	$x4 = 0.618691$	$x3 = 0.750000$	$y2 = 5,516.007425$	$c3 = 1.009230$
	$x5 = 0.090000$	$x5 = 0.030000$	$x5 = 0.030000$	$x5 = 0.040000$	$x4 = 0.156233$	$y3 = 0.727879$	$c4 = 1.008593$
	$x6 = 30000.00$	$x6 = 54,000.00$	$x6 = 33,000.00$	$x6 = 58,000.00$	$x5 = 0.60000$	$y5 = 19,177.336319$	$c5 = 1.007833$
	$x7 = 1.700000$	$x7 = 1.600000$	$x7 = 1.600000$	$x7 = 1.500000$	$x6 = 60,000.00$	$y6 = 0.960301$	$c6 = 0.960301$
	$x8 = 7.500000$	$x8 = 5.400000$	$x8 = 4.300000$	$x8 = 4.400000$	$x7 = 1.400000$	$y7 = 44,756.107036$	$c7 = 1.040000$
	$x9 = 43.00000$	$x9 = 48.00000$	$x9 = 67.00000$	$x9 = 57.00000$	$x8 = 2.500000$	$y8 = 8.113859$	$c8 = 0.727879$
	$x10 = 1,330.00$	$x10 = 1,150.00$	$x10 = 550.000$	$x10 = 670.000$	$x9 = 70.00000$	$y9 = 0.923939$	$c9 = 0.836745$
	$f1 = 196.460454$		$f2 = 740.752732$		$x10 = 1,490.00$	$y10 = 9,370.659256$	$c10 = -0.78\mathrm{e}^{-3}$
	$f3 = 383.635843$		$f4 = 1,060.499087$				
6	$x1 = 0.223930$	$x1 = 0.214867$	$x1 = 0.253804$	$x1 = 0.192584$			
	$x2 = 0.900000$	$x2 = 1.050000$	$x2 = 1.100000$	$x2 = 1.100000$	$x1 = 0.114020$		$c1 = 1.002356$
	$x3 = 1.100000$	$x3 = 1.150000$	$x3 = 1.250000$	$x3 = 1.000000$	$x2 = 0.750000$	$y1 = 44,613.135730$	$c2 = 1.008139$
	$x4 = 0.519545$	$x4 = 0.498969$	$x4 = 0.868203$	$x4 = 0.567149$	$x3 = 0.750000$	$y2 = 5,479.032162$	$c3 = 1.008567$
	$x5 = 0.090000$	$x5 = 0.040000$	$x5 = 0.070000$	$x5 = 0.060000$	$x4 = 0.155672$	$y3 = 0.725604$	$c4 = 1.008036$
	$x6 = 43,000.00$	$x6 = 49,000.00$	$x6 = 41,000.00$	$x6 = 37,000.00$	$x5 = 0.060000$	$y5 = 19,004.700585$	$c5 = 1.007353$
	$x7 = 1.800000$	$x7 = 1.600000$	$x7 = 1.800000$	$x7 = 1.500000$	$x6 = 60,000.00$	$y6 = 0.961383$	$c6 = 0.961383$
	$x8 = 3.800000$	$x8 = 2.800000$	$x8 = 2.900000$	$x8 = 8.000000$	$x7 = 1.400000$	$y7 = 44,613.135730$	$c7 = 1.040000$
	$x9 = 49.00000$	$x9 = 57.00000$	$x9 = 40.00000$	$x9 = 45.000000$	$x8 = 2.500000$	$y8 = 8.142521$	$c8 = 0.725604$
	$x10 = 750.000$	$x10 = 940.000$	$x10 = 1,410.00$	$x10 = 1,410.00$	$x9 = 70.00000$	$y9 = 0.924202$	$c9 = 0.836745$
	$f1 = 397.865281$		$f2 = 800.618592$		$x10 = 1,480.00$	$y10 = 9,339.898473$	$c10 = -0.3665\mathrm{e}^{-2}$
	$f3 = 374.696788$		$f4 = 300.074226$				
7	$x1 = 0.186602$	$x1 = 0.300930$	$x1 = 0.248547$	$x1 = 0.167753$			
	$x2 = 0.900000$	$x2 = 0.950000$	$x2 = 1.050000$	$x2 = 1.200000$	$x1 = 0.112264$		$c1 = 1.003305$
	$x3 = 0.850000$	$x3 = 0.800000$	$x3 = 0.850000$	$x3 = 1.100000$	$x2 = 0.750000$	$y1 = 44,933.291470$	$c2 = 1.009043$
	$x4 = 0.117502$	$x4 = 0.473359$	$x4 = 0.859795$	$x4 = 0.502709$	$x3 = 0.750000$	$y2 = 5,552.987948$	$c3 = 1.009347$
	$x5 = 0.060000$	$x5 = 0.070000$	$x5 = 0.040000$	$x5 = 0.020000$	$x4 = 0.156241$	$y3 = 0.732720$	$c4 = 1.008709$
	$x6 = 53,000.00$	$x6 = 57,000.00$	$x6 = 49,000.00$	$x6 = 45,000.00$	$x5 = 0.060000$	$y5 = 19,350.552345$	$c5 = 1.007941$
	$x7 = 1.500000$	$x7 = 1.500000$	$x7 = 1.400000$	$x7 = 1.500000$	$x6 = 60,000.00$	$y6 = 0.960004$	$c6 = 0.960004$
	$x8 = 2.800000$	$x8 = 6.200000$	$x8 = 4.000000$	$x8 = 2.500000$	$x7 = 1.400000$	$y7 = 44,933.291470$	$c7 = 1.040000$
	$x9 = 48.00000$	$x9 = 44.00000$	$x9 = 42.00000$	$x9 = 58.00000$	$x8 = 2.500000$	$y8 = 8.091732$	$c8 = 0.732720$
	$x10 = 960.000$	$x10 = 1,170.00$	$x10 = 1,130.00$	$x10 = 960.000$	$x9 = 70.00000$	$y9 = 0.923935$	$c9 = 0.836745$
	$f1 = 873.735778$		$f2 = 874.042044$		$x10 = 1,500.00$	$y10 = 9,436.104076$	$c10 = -0.24\mathrm{e}^{-4}$
	$f3 = 472.332724$		$f4 = 571.864600$				
8	$x1 = 0.309000$	$x1 = 0.135044$	$x1 = 0.290600$	$x1 = 0.281596$			
	$x2 = 1.100000$	$x2 = 0.800000$	$x2 = 0.950000$	$x2 = 0.800000$	$x1 = 0.165940$		$c1 = 0.995067$
	$x3 = 1.100000$	$x3 = 1.250000$	$x3 = 1.150000$	$x3 = 1.100000$	$x2 = 0.800000$	$y1 = 48,989.982126$	$c2 = 0.999959$
	$x4 = 0.600457$	$x4 = 0.195630$	$x4 = 0.585490$	$x4 = 0.933800$	$x3 = 0.750000$	$y2 = 5,443.692360$	$c3 = 1.001187$
	$x5 = 0.080000$	$x5 = 0.04000$	$x5 = 0.020000$	$x5 = 0.020000$	$x4 = 0.156228$	$y3 = 0.718359$	$c4 = 1.001534$
	$x6 = 31,000.00$	$x6 = 55,000.00$	$x6 = 42,000.00$	$x6 = 31,000.00$	$x5 = 0.060000$	$y5 = 18,832.647098$	$c5 = 1.001603$
	$x7 = 1.600000$	$x7 = 1.800000$	$x7 = 1.800000$	$x7 = 1.700000$	$x6 = 60000.00$	$y6 = 0.963637$	$c6 = 0.963637$
	$x8 = 7.200000$	$x8 = 8.200000$	$x8 = 7.800000$	$x8 = 3.300000$	$x7 = 1.400000$	$y7 = 48,989.982126$	$c7 = 1.040000$
	$x9 = 57.00000$	$x9 = 51.00000$	$x9 = 59.00000$	$x9 = 70.000000$	$x8 = 2.500000$	$y8 = 8.999403$	$c8 = 0.718359$
	$x10 = 1,400.000$	$x10 = 1,120.00$	$x10 = 540.000$	$x10 = 1,260.000$	$x9 = 70.00000$	$y9 = 0.923941$	$c9 = 0.836745$
	$f1 = 290.574850$		$f2 = 508.127100$		$x10 = 1,470.00$	$y10 = 9,242.005348$	$c10 = -0.108\mathrm{e}^{-3}$
	$f3 = 405.687547$		$f4 = 250.653220$				

[a]All corresponding values of range* ("*" indicates the optimum solution) are listed in Table 14.
[b]In the column of "Initial infeasible experimental points," each column from $x1$ to $x10$ represents an initial experimental point.

active design variable control dynamically and effectively controls the number of active design variable, so as to reduce the dimension of the problem. The sensitivity information used in the active design variable control is a byproduct of the optimization process with no extra costs. In solving the conceptual aircraft design problem, the ECPM is competitively efficient and it readily deals with constraints in comparison with the BLISS. However, the BLISS has a better accuracy than the ECPM for solving the conceptual aircraft design problem. As the number of variables grows, more random samples are required to effectively cover the design variable space. High computational capacity, such as parallel computing, may facilitate the ECPM for solving large MDO problems.

Appendix: Results of the Conceptual Aircraft Optimization

The following conditions hold for Table A1: $x1 = \lambda$; $x2 = x$; $x3 = C_f$; $x4 = T$; $x5 = t/c$; $x6 = h$; $x7 = M$; $x8 = \mathrm{AR}$; $x9 = \Lambda$; $x10 = S_{\mathrm{REF}}$; $f = \mathrm{range}$; $y1 = W_T$; $y2 = D$; $y3 = \mathrm{ESF}$; $y5 = W_F$; $y6 = \Theta$; $y7 = L$; $y8 = L/D$; $y9 = \mathrm{SFC}$; $y10 = W_E$; $c1 = \sigma_1$; $c2 = \sigma_2$; $c3 = \sigma_3$; $c4 = \sigma_4$; $c5 = \sigma_5$; $c6 = \Theta$; $c7 = \mathrm{d}p/\mathrm{d}x$; $c8 = \mathrm{ESF}$; $c9 = \mathrm{Temp}$; $c10 = \bar{T} - \bar{T}_{\mathrm{UA}}$.

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